Lecture 1

GNSS measurements and their combinations

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Contents

1. Review of GNSS measurements.
2. Linear combinations of measurements.
3. Carrier cycle-slips detection.
4. Carrier smoothing of code pseudorange.
1. Review of GNSS measurements.
2. Linear combinations of measurements.
3. Carrier cycle-slips detection.
4. Carrier smoothing of code pseudorange.
GPS SIGNAL STRUCTURE

Two carriers in L-band:
- \( L_1 = 154 \) \( f_0 = 1575.42 \) MHz
- \( L_2 = 120 \) \( f_0 = 1227.60 \) MHz

where \( f_0 = 10.23 \) MHz

- C/A-code for civilian users \([X_C(t)]\)
- P-code only for military and authorized users \([X_P(t)]\)
- Navigation message with satellite ephemeris and clock corrections \([D(t)]\)

\[
S^{(k)}_{L_1}(t) = a_P X^{(k)}_P(t) D^{(k)}(t) \sin(\omega_1 t + \phi_{L_1}) + a_C X^{(k)}_C(t) D^{(k)}(t) \cos(\omega_1 t + \phi_{L_1})
\]

\[
S^{(k)}_{L_2}(t) = b_P X^{(k)}_P(t) D^{(k)}(t) \sin(\omega_2 t + \phi_{L_2})
\]
From hereafter we will call:

- **$C_1$** pseudorange computed from $X_C(t)$ binary code (on frequency 1)
- **$P_1$** pseudorange computed from $X_P(t)$ binary code (on frequency 1)
- **$P_2$** pseudorange computed from $X_P(t)$ binary code (on frequency 2)
GPS Carrier Phase Measurements

\[ S_{L_1}^{(k)}(t) = a_p X_P^{(k)}(t) D^{(k)}(t) \sin(\omega t + \varphi_{L_1}) + a_c X_C^{(k)}(t) D^{(k)}(t) \cos(\omega t + \varphi_{L_1}) \]
\[ S_{L_2}^{(k)}(t) = b_p X_P^{(k)}(t) D^{(k)}(t) \sin(\omega t + \varphi_{L_2}) \]

Carrier beat phase:

\[ \phi_L(T) = \phi_{L_{rec}}(T) - \phi_{L_{sat}}(T - \Delta \tilde{T}) \]
\[ = \frac{c}{\lambda} \Delta \tilde{T} + N \]  
Unknown ambiguity

From hereafter we will call:

- \( L_1 = \lambda_1 \phi_{L_1} \) measur. computed from the carrier phase on frequency 1
- \( L_2 = \lambda_2 \phi_{L_2} \) measur. computed from the carrier phase on frequency 2
- \( C_1 \) pseudorange computed from \( X_C(t) \) binary code (on frequency 1)
- \( P_1 \) pseudorange computed from \( X_P(t) \) binary code (on frequency 1)
- \( P_2 \) pseudorange computed from \( X_P(t) \) binary code (on frequency 2)
Carrier and Code pseudorange measurement

\[ P_1 = c \Delta T = c [t_{\text{rec}}(T) - t_{\text{sat}}(T-\Delta T)] \]

\[ P_1 \approx \rho + \text{clock offset} \approx 20,000 \text{Km} \]

\( P_1 \) is basically the geometric range (\( \rho \)) between satellite and receiver, plus the relative clock offset. The range varies in time due to the satellite motion relative to the receiver.

\( P_1 \) is an absolute measurement (unambiguous)
Phase and Code pseudorange measurements

\[ L_1(T) = c \Delta \tilde{T} + \lambda_1 N_1 \]

Relative measurement (shifted by the unknown ambiguity \( \lambda N \))

Each time that the receiver loose the phase lock, the unknown ambiguity changes by an integer number of \( \lambda \)

\[ L_1 \approx \rho + \text{clock offset} + \lambda_1 N_1 \]
Code and Carrier Phase measurements

- **Code** (unambiguous but noisier)
- **Carrier Phase** (ambiguous but precise)

Ionospheric combination (meters, PRN01)

UT (seconds, 1997 January 9th)
**GPS measurements: Code and Carrier Phase**

**Antispoofing (A/S):**
The code P is encrypted to Y. ➔ Only the code C at frequency L1 is available.

<table>
<thead>
<tr>
<th>Wavelength (chip-length)</th>
<th>σ noise (1% of λ) [*]</th>
<th>Main characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Code measurements</strong></td>
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<td></td>
</tr>
<tr>
<td>C₁</td>
<td>300 m</td>
<td>3 m</td>
</tr>
<tr>
<td>P₁ (Y1): encrypted</td>
<td>30 m</td>
<td>30 cm</td>
</tr>
<tr>
<td>P₂ (Y2): encrypted</td>
<td>30 m</td>
<td>30 cm</td>
</tr>
<tr>
<td><strong>Phase measurements</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L₁</td>
<td>19.05 cm</td>
<td>2 mm</td>
</tr>
<tr>
<td>L₂</td>
<td>24.45 cm</td>
<td>2 mm</td>
</tr>
</tbody>
</table>

[*] the codes can be smoothed with the phases in order to reduce noise (i.e, C₁ smoothed with L₁ ➔ 50 cm noise)
RINEX FILES
Pseudoranges (C/A, P1, P2), phase tracking (L1, L2)
Navigation data (D(t))

One or multiple antennas

Low-noise Amplifiers

Low-noise Amplifiers

RF/IF & SAMPLING

1

DLL Tracking RCVR
Demodulator

Navigation data processing &
Pseudorange correction

Kalman filter position estimation

DISPLAY

Man-Machine Interface

Aiding or integrated receiver

EXTERNAL SENSORS

Master of Science in GNSS
@ J. Sanz & J.M. Juan
GNSS data files follow a well defined set of standards formats: RINEX, ANTEX, SINEX...

Understanding a format description is a tough task.

These standards are explained in a very easy and friendly way through a set of html files.

Described formats:
- Observation RINEX
- Navigation RINEX
- RINEX CLOCKS
- SP3 Version C
- ANTEX

Open GNSS Formats with Firefox internet browser

More details at: http://www.gage.es/gLAB
### RINEX measurement file

**HEADER**

```
2 OBSERVATION DATA G (GPS) RINEX VERSION / TYPE
RGRINEXO V2.4.1 UX AUSLIG 10-JAN-97 10:19 PGM / RUN BY / DATE
Australian Regional GPS Network (ARGN) - Cocos Island COMMENT
BIT 2 OF LLI (+4) FLAGS DATA COLLECTED UNDER "AS" CONDITION COMMENT
-0.0000000000103 HARDWARE CALIBRATION (S) COMMENT
-0.000000054663 CLOCK OFFSET (S) COMMENT
COCO MARKER NAME
AU18 MARKER NUMBER
MRH OBSERVER / AGENCY
126 MARGUE SNR-8100 REC # / TYPE / VERS
327 DÖRNE MARGOLIN T ANT # / TYPE
-741950.3241 6190961.9624 -1337769.9813 APPROX POSITION XYZ
0.0040 0.0000 0.0000 ANTENNA: DELTA H/E/N
1 1 WAVELENGTH FACT L1/L2
5 C1 L1 L2 P2 P1 # / TYPES OF OBSERV
SNR is mapped to signal strength [0,1,4-9] COMMENT
SNR: >500 >100 >50 >10 >5 >0 bad n/a COMMENT
sig: 9 8 7 6 5 4 1 0 COMMENT
30 INTERVAL
1997 1 9 0 7 30.00000000 TIME OF FIRST OBS
1997 1 9 23 59 30.00000000 TIME OF LAST OBS
```

**MEASUREMENTS**

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<th>M/3</th>
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</table>
```
### RINEX Measurement File

**RINEX Version / Type:**
- PGM / RUN BY / DATE
- COMMENT

**Australian Regional GPS Network (ARGN) - COCOS ISLAND**
- BIT 2 OF LLI (+4) FLAGS DATA COLLECTED UNDER "AS" CONDITION
- HARDWARE CALIBRATION (S)
- CLOCK OFFSET (S)

**Marker Name:**
- MARKER NAME
- MARKER NUMBER

**Observer / Agency:**
- OBSERVER / AGENCY

**Internal Position XYZ:**
- INTERNAL DELTA X/Y/Z
- WAVELENGTH FACT L1/L2
- # / TYPES OF OBSERV

**SNR is mapped to signal strength [0,1,4-9]:**
- SNR: >500 >100 >50 >10 >5 >0 bad n/a
- sig: 9 8 7 6 5 4 1 0

**Time of First Obs:**
- TIME OF FIRST OBS

**Time of Last Obs:**
- TIME OF LAST OBS

**End of Header:**

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RI NEX measurement file

2 OBSERVATION DATA G (GPS)
RGRINEXO V2.4.1 UX AUSLIG 10-JAN-97 10:19
Australian Regional GPS Network (ARGN) - COCOS ISLAND
BIT 2 OF LLI (+4) FLAGS DATA COLLECTED UNDER "AS" CONDITION
-0.000000000103 HARDWARE CALIBRATION (S)
-0.000000054663 CLOCK OFFSET (S)
COCO
AU18
mhr
5 11 1 L1 L2 P2 P1
SNR is mapped to signal strength [0-1.4-9]
SNR: 500 >100
sig: 9 8
1997 1 9 0 7 30.00000000
1997 1 9 23 59 30.00000000
97 1 9 0 7 30.00000000
5 1 25 9 5 23 17 6
22127685.105 -14268715.360 -11118481.284 22127685.4014 <= 1
22672158.746 -11510817.289 -8969469.3004 22672158.5184 <= 25
22594902.367 -12949753.825 -10090708.5394 22594903.7394 <= 9
22731128.796 -1162184.951 -9055464.1694 22731130.0094 <= 5
24610920.702 -924108.174 -720085.7045 24610950.4144 <= 23
20718299999999 -14498133.3 -14870090.5568 20842713.4014 <= 17
20842713.4014 9732678.105 -14268715.360 -11118481.284 22127685.4014 <= 6

Measurement time (receive time tags)
Number of tracked satellites
Epoch flag 0: OK
One satellite per row
RI NEX measurement file

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</table>

**Synthetic P2 (A/S=on)**

**S/ N indicator**

**Loss of lock indicator**
P = c \Delta T = c \left[ t_{\text{rec}}(T) - t_{\text{sat}}(T - \Delta T) \right]

P_{\text{sat}}^{\text{rec}} = \rho_{\text{rec}}^{\text{sat}} + c \cdot (dt_{\text{rec}} - dt_{\text{sat}}) + \sum \delta

\sum \delta = Trop_{\text{rec}}^{\text{sat}} + Ion_{\text{rec}}^{\text{sat}} + K_{\text{rec}} + K_{\text{sat}}^{\text{sat}} + \varepsilon

Geometric range

Clock offsets

Tropospheric delay

Ionospheric delay

Instrumental delays

noise
Pseudorange

Geometric range: $\rho \approx 20,000 \text{ km}$

Satellite clock offset
up to hundreds of km

Relativistic clock correction
$< 13 \text{ m}$

Satellite instrumental delay
$\approx \text{ m}$

Ionospheric delay
$2 - 30 \text{ m}$

Tropospheric delay
$2 - 30 \text{ m}$

Receiver clock offset
$< 300 \text{ km}$

Receiver instrumental delay
$\approx \text{ m}$

$\Delta t$
Pseudorange

Geometric range: $\rho \sim 20,000$ km

Satellite clock offset
up to hundreds of km

Relativistic clock correction: $< 13$ m

Satellite instrumental delay $\sim$ m

Ionospheric delay: $2 - 30$ m

Tropospheric delay: $2 - 30$ m

Receiver clock offset: $< 300$ km

Receiver instrumental delay: $\sim$ m

~300m

Emission

Reception
Exercise:

a) Using the file 95oct18casa___r0.rnx, generate the “txt” file 95oct18casa.a (with data ordered in columns).

b) Plot code and phase measurements for satellite PRN28 and discuss the results.

Resolution:

a) \texttt{gLAB\_linux} -input:cfg \texttt{meas.cfg} -input:obs \texttt{coco0090.97o}

b) See next plots:
An example of program to read the RINEX: **gLAB**

The RINEX file is converted to a “columnar format” to easily plot its contents and to analyze the measurements (the public domain free tool **gnuplot** is used in the book to make the plots).

**RINEX file ➔ gLAB ➔ txt file**
The geometry \( \rho \) is the dominant term in the plot. The pattern in the figures is due to the variation of \( \rho \).
Similar plot for code measurements at $f_2$. Notice that

- Ionosphere ($\text{Ion}$) and
- Instrumental delays ($K$) depend on frequency.

$$P_{2\text{sta}}^{\text{sat}} = \rho_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}} - dt_{\text{sat}}) + T_{\text{sta}}^{\text{sat}} + Ion_{2\text{sta}}^{\text{sat}} + K_{2\text{sta}} + K^{\text{sat}}_2 + \epsilon_2$$
**Code measurements: C1,P1,P2**

\[
P_{1^{\text{sat}}_{\text{sta}}} = \rho_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}}^{\text{sat}} - dt_{\text{sta}}) + Trop_{\text{sta}}^{\text{sat}} + Ion_{\text{sta}}^{\text{sat}} + K_{1^{\text{sta}}}^{\text{1}} + K_{2^{\text{sta}}}^{\text{2}} + \varepsilon_1
\]

\[
P_{2^{\text{sat}}_{\text{sta}}} = \rho_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}}^{\text{sat}} - dt_{\text{sta}}) + Trop_{\text{sta}}^{\text{sat}} + Ion_{\text{sta}}^{\text{sat}} + K_{1^{\text{sta}}}^{\text{2}} + K_{2^{\text{sta}}}^{\text{2}} + \varepsilon_2
\]

**Phase measurements: L1,L2**

\[
L_{1^{\text{sat}}_{\text{sta}}} = \rho_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}}^{\text{sat}} - dt_{\text{sta}}) + Trop_{\text{sta}}^{\text{sat}} - Ion_{1^{\text{sta}}}^{\text{sat}} + b_{1^{\text{sta}}}^{\text{sat}} + b_{1}^{\text{sat}} + \lambda_{1}N_{1} + \lambda_{1}w + \nu_{1}
\]

\[
L_{2^{\text{sat}}_{\text{sta}}} = \rho_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}}^{\text{sat}} - dt_{\text{sta}}) + Trop_{\text{sta}}^{\text{sat}} - Ion_{2^{\text{sta}}}^{\text{sat}} + b_{2^{\text{sta}}}^{\text{sat}} + b_{2}^{\text{sat}} + \lambda_{2}N_{2} + \lambda_{2}w + \nu_{2}
\]

**Phase Ambiguities**

\[N_1, N_2 \text{ are integers}\]
Carrier Phase measurements

The geometry “\( \rho \)” is the dominant term in the plot. The pattern in the figures is due to the variation of “\( \rho \)”.

The curves are broken when the receiver loss the lock (cycle-slip).

When a cycle-slip happens, the phase measurement “\( L \)” changes by an unknown integer number of cycles (\( N \))

\[
L_{sta}^{sat} = \rho_{sta}^{sat} + c \cdot (dt_{sta}^{sat} - dt_{sat}^{sat}) + Trop_{sta}^{sat} - Ion_{sta}^{sat} + b_{sta} + b_{sat} + \lambda_1 N_1 + \lambda_1 w + \nu_1
\]
When a cycle-slip happens, the phase measurement "$L$" changes by an unknown integer number of cycles ($N$).

The geometry "$\rho$" is the dominant term in the plot. The pattern in the figures is due to the variation of "$\rho$".

The curves are broken when the receiver loses the lock (cycle-slip).
Contents

1. Review of GNSS measurements.
2. Linear combinations of measurements.
3. Carrier cycle-slips detection.
4. Carrier smoothing of code pseudorange.
Linear Combinations of measurements:

- Geometry-free (or Ionospheric) combination.
- Ionosphere-Free combination.
- Wide-lane and Narrow-lane combinations.
1. Geometry-free (or ionospheric) combination

\[ P_1 = P_2 - P_1 = \text{Iono} + ctt \]

\[ L_1 = L_1 - L_2 = \text{Iono} + ctt + \text{Ambig} \]

**Code measurements:** \( C_1, P_1, P_2 \)

\[
P_{1\text{sta}}^{\text{sat}} = o_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}} - dt_{\text{sat}}) + Trop_{\text{sta}}^{\text{sat}} + \text{Ion}_{1\text{sta}}^{\text{sat}} + K_{1\text{sta}} + K_{1\text{sat}}^{\text{sat}} + \epsilon_1
\]

\[
P_{2\text{sta}}^{\text{sat}} = o_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}} - dt_{\text{sat}}) + Trop_{\text{sta}}^{\text{sat}} + \text{Ion}_{2\text{sta}}^{\text{sat}} + K_{2\text{sta}} + K_{2\text{sat}}^{\text{sat}} + \epsilon_2
\]

**Carrier measurements:** \( L_1, L_2 \)

\[
L_{1\text{sta}}^{\text{sat}} = o_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}} - dt_{\text{sat}}) + Trop_{\text{sta}}^{\text{sat}} - \text{Ion}_{1\text{sta}}^{\text{sat}} + b_{1\text{sta}} + b_{1\text{sat}} + \lambda_1 N_1 + \lambda_1 w + v_1
\]

\[
L_{2\text{sta}}^{\text{sat}} = o_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}} - dt_{\text{sat}}) + Trop_{\text{sta}}^{\text{sat}} - \text{Ion}_{2\text{sta}}^{\text{sat}} + b_{2\text{sta}} + b_{2\text{sat}} + \lambda_2 N_2 + \lambda_2 w + v_2
\]

Master of Science in GNSS
1. Geometry-free (or ionospheric) combination

\[ P_1 = P_2 - P_1 = \text{Iono} + ctt \]

\[ L_1 = L_1 - L_2 = \text{Iono} + ctt + \text{Ambig} \]

- The pattern corresponds to the ionospheric refraction (\( \text{Ion} \)), because the other terms (\( K \)) are constant.
- Notice that code measurements are noisier.
Ionospheric effects

The ionospheric refraction depends on:
- Geographic location
- Time of day
- Time with respect to solar cycle (11y)
The ionospheric delay ($Ion$) is proportional to the electron density integrated along the ray path ($STEC$).

$$Ion = \frac{40.3}{f^2} STEC$$

$$STEC = \int \frac{\bar{r}(GPSreceiver)}{\bar{r}(GPStransmitter)} N_e(\bar{r}, t) dr$$
The ionospheric refraction depends on the inverse of the squared frequency and can be removed up to 99.9% combining \( f_1 \) and \( f_2 \) signals:

\[
P_c = \frac{f_1^2 P_1 - f_2^2 P_2}{f_1^2 - f_2^2}
\]

\[
L_c = \frac{f_1^2 L_1 - f_2^2 L_2}{f_1^2 - f_2^2}
\]

\[
P_{c,\text{sat}}^{\text{sta}} = \rho_{\text{sta}}^{\text{sat}} + c \cdot (d_{t,\text{sta}} - d_{t,\text{sat}}) + \text{Trop}_{\text{sta}}^{\text{sat}} + \varepsilon_c
\]

\[
L_{c,\text{sat}}^{\text{sta}} = \rho_{\text{sta}}^{\text{sat}} + c \cdot (d_{t,\text{sta}} - d_{t,\text{sat}}) + \text{Trop}_{\text{sta}}^{\text{sat}} + b_{c,\text{sta}} + b_{c}^{\text{sat}} + \lambda_N R_C + \lambda_N w + v_c
\]

- The ionospheric refraction has been removed in \( L_c \) and \( P_c \)

\( \lambda_N = 10.7 \text{ cm, } \lambda_w = 86.2 \text{ cm} \)

The \( R_C \) ambiguities are NOT integers!!

\[
R_C = N_1 - \frac{\lambda_w}{\lambda_2} (N_1 - N_2)
\]
Comments:

Two-frequency receivers are needed to apply the ionosphere-free combination.

If a one-frequency receiver is used, an ionospheric model must be applied to remove the ionospheric refraction. The GPS navigation message provides the parameters of the Klobuchar model which accounts for more than 50% (RMS) of the ionospheric delay.
3.- Narrow-lane ($P_N$) and Wide-lane Combination ($L_W$)

The wide-lane combination $L_W$ provides a signal with a large wavelength ($\lambda_W = 86.2\text{cm} \sim 4\lambda_1$). This makes it very useful for detecting cycle-slips through the Melbourne-Wübbenena combination: $L_W - P_N$

$$P_N = \frac{f_1 P_1 + f_2 P_2}{f_1 + f_2}$$

$$L_W = \frac{f_1 L_1 - f_2 L_2}{f_1 - f_2}$$

The ambiguities $N_W$ are integers!

The same sign

No wind-up
Exercises:

1) Consider the wide-lane combination of carrier phase measurements

\[ L_W = \frac{f_1 L_1 - f_2 L_2}{f_1 - f_2} \], where \( L_W \) is given in length units (i.e. \( L_i = \lambda_i \phi_i \)).

Show that the corresponding wavelength is:

\[ \lambda_W = \frac{c}{f_1 - f_2} \]

**Hint:**

\[ L_W = \lambda_W \phi_W \; ; \; \phi_W = \phi_1 - \phi_2 \]

2) Assuming \( L_1, L_2 \) uncorrelated measurements with equal noise \( \sigma_L \), show that:

\[ \sigma_{L_W} = \frac{\sqrt{\gamma_{12}} + 1}{\sqrt{\gamma_{12}} - 1} \sigma_L \; ; \; \gamma_{12} = \left( \frac{f_1}{f_2} \right)^2 \]
Contents

1. Review of GNSS measurements.
2. Linear combinations of measurements.
3. Carrier cycle-slips detection.
4. Carrier smoothing of code pseudorange.
Detecting cycle-slips

This cycle-slip involves millions of cycles ➔ it is easy to detect!!

There is a cycle-slip of only one cycle (~20cm) ➔ How to detect it?

@ J. Sanz & J. M. Juan
Exercise:

a) Using the file 95oct18casa___r0.rnx, generate the “txt” file 95oct18casa.a (with data ordered in columns).

b) Insert a cycle-slip of “one wavelength” (19cm) in L1 measurement at t=5000 s (and no cycle-slip in L2).

c) Plot the measurements “L1, L1-P1, LC-PC, Lw-PN and L1-L2” and discuss which combination/s should be used to detect the cycle-slip.

Resolution:

a) gLAB_linux -input:cfg meas.cfg -input:obs 95oct18casa_r0.rnx

b) cat 95oct18casa.a | gawk ‘{if ($4==18) print $3,$5,$6,$7,$8}’ > s18.org

  cat s18.org | gawk ‘{if ($1>=5000) $2=$2+0.19; printf “%s %f %f %f %f %f \n”, $1,$2,$3,$4,$5}’ > s18.cl

c) See next plots:
The geometry \( \rho \) is the dominant term in the plot. The variation of \( \rho \) in 1 sec may be hundreds of meters, many times greater than the cycle-slip (19 cm) \( \Rightarrow \) the variation of \( \rho \) shadows the cycle-slip!

\[
L_{\text{sat}}^{\text{sta}} = \rho_{\text{sat}}^{\text{sta}} + c \cdot (d_{\text{sta}}^{\text{sat}} - d_{\text{sta}}^{\text{sat}}) + Trop_{\text{sta}}^{\text{sat}} - Ion_{\text{1sta}}^{\text{sat}} + b_{\text{1sta}} + b_{1}^{\text{sat}} + \lambda_{1} N_{1} + \lambda_{1} w + v_{1}
\]
The geometry and clock offsets have been removed. The trend is due to the Ionosphere. The \( P1 \) code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.

A jump of \( \lambda = 19 \text{ cm} \) (one cycle in L1) has been introduced in L1 at \( t = 5000s \).

\[
L_{1\text{sta}} - P_{1\text{sta}} = -2Ion_{1\text{sta}}^{\text{sat}} + ctt + \text{ambig} + \varepsilon
\]
The geometry and clock offsets have been removed. The trend is due to the Ionosphere. The P1 code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.

A jump of $\lambda = 19 \text{ cm}$ (one cycle in L1) has been introduced in L1 at $t=5000s$.

**Equations:**

\[ L_{sat}^1 - P_{sat}^1 = -2 \text{Ion}_{sat}^1 + \text{ctt} + \text{ambig} + \varepsilon \]

\[ P_{sat}^1 = \rho_{sat}^1 + c \cdot (dt_{sta} - dt_{sat}) + \text{Trop}_{sta}^1 + \text{Ion}_{sat}^1 + K_{1sta} + K_{1sat} + \varepsilon_{1}^{50} \]

\[ L_{sat}^1 = \rho_{sat}^1 + c \cdot (dt_{sta} - dt_{sat}) + \text{Trop}_{sta}^1 - \text{Ion}_{sat}^1 + b_{1sta} + b_{1sat} + \lambda_{1} N_{1} + \lambda_{1} w + v_{1} \]
The geometry, clock offsets and iono have been removed. There is a constant pattern plus noise. The $P1$ code noise also shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.

A jump of $\lambda=19\text{ cm (one cycle in L1)}$ has been introduced in $L1$ at $t=5000s$.

\[
L_{c,sta}^{sat} - P_{c,sta}^{sat} = ctt + ambig + \varepsilon
\]
The geometry, clock offsets and iono have been removed. There is a constant pattern plus noise. The P1 code noise also shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.

A jump of $\lambda = 19 \text{ cm}$ (one cycle in L1) has been introduced in L1 at $t=5000s$.

$LI_{sta}^{sat} - PI_{sta}^{sat} = ctt + \text{ambig} + \epsilon$

$PI_{sta}^{sat} = lon_L + K_{sta} + K_{sat}^{L1} + \epsilon_L$

$LI_{sta}^{sat} = lon_L + b_{sta}^{L1} + b_{sat}^{L1} + \lambda_1 N_1 - \lambda_2 N_2 + (\lambda_1 - \lambda_2) w + \nu_L$
The geometry, clock offsets and iono have been removed. There is a constant pattern plus noise. The \( P_N \) code noise is under one cycle of \( L_w \). Thence, the cycle-slip is clearly detected.

A jump of \( \lambda = 19 \text{ cm} \) (one cycle in L1) has been introduced in L1 at \( t=5000s \).

The equation is:

\[
L_{\text{w,sta}}^{\text{sat}} - P_{\text{n,sta}}^{\text{sat}} = ctt + \text{ambig} + \epsilon
\]

\[
P_{\text{n,sta}}^{\text{sat}} = \rho_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}} - dt_{\text{sat}}) + Trop_{\text{sta}}^{\text{sat}} + \text{Ion}_{\text{w,sta}}^{\text{sat}} + K_{\text{w,sta}}^{\text{sat}} + \epsilon_{\text{N}}
\]

\[
L_{\text{w,sta}}^{\text{sat}} = \rho_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}} - dt_{\text{sat}}) + Trop_{\text{sta}}^{\text{sat}} + \text{Ion}_{\text{w,sta}}^{\text{sat}} + \lambda_{\text{w,sta}}^{\text{sat}} + \epsilon_{\text{N}}
\]
The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_1$ code noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

A jump of $\lambda = 19$ cm (one cycle in $L_1$) has been introduced in $L_1$ at $t=5000s$. The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_1$ code noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.
Summary

Session 3b, exercise 2b: Cycle-slip detection with L1-P1, PRN 18

L1

L1-P1

LI-PI

LC-PC

Master of Science in GNSS
The cycle-slips are detected by the Ionospheric combination (LI=L1-L2) and the Melbourne Wübbena (W=Lw-PN)

Two independent combinations, LI and Lw, allow to detect two independent cycle-slips (in L1 and L2 phase measur.).

Notice that, from L1, L2 is not possible to detect short cycle-slips.
Contents

1. Review of GNSS measurements.
2. Linear combinations of measurements.
3. Carrier cycle-slips detection.
   3.1 Cycle-slip Detection Algorithms
4. Carrier smoothing of code pseudorange.
Cycle-slip detector based on carrier phase data: The Geometry-free combination

Input data: Geometry-free combination of carrier phase measurements

\[ L_I = L_1(s; k) - L_2(s; k) \]

Output: [satellite, time, cycle-slip flag].

For each epoch \(k\)

For each tracked satellite \(s\)

- Declare cycle slip when data gap greater than \(tol_{\Delta t}\).\(^{16}\)
- Fit a second-degree polynomial \(p(s; k)\) to the previous values (after the last cycle-slip) \([L_I(s; k - N_I), \ldots, L_I(s; k - 1)]\).
- Compare the measured \(L_I(s; k)\) and the predicted value \(p(s; k)\) at epoch \(k\). If the discrepancy exceeds a given threshold, then declare cycle slip. That is,

\[
\text{if } |L_I(s; k) - p(s; k)| > \text{threshold} \text{ then cycle slip.}
\]

- Reset algorithm after cycle slip.

End

End
Under not disturbed ionospheric conditions, the geometry-free combination performs as a very precise and smooth test signal, driven by the ionospheric refraction. Although, for instance, the jump produced by a simultaneous one-cycle slip in both signals is smaller in this combination than in the original signals ($\lambda_2 - \lambda_1 = 5.4\text{cm}$), it can provide reliable detection even for small jumps.
Cycle-slip detector based on code and carrier phase data: The Melbourne-Wübbena combination

Input data: Melbourne-Wübbena combination

Output: [satellite (PRN), time, cycle-slip flag]

For each epoch \((k)\):

- Declare cycle-slip when data hole greater than \(\text{tol}_{\Delta t}\) (e.g., 60 s).
- If no data hole larger than \(\text{tol}_{\Delta t}\), thence:
- Compare the measurement \(B_W(s; k)\) at the epoch \(k\) with the mean bias \(m_{B_W}(s; k - 1)\) computed from the previous values. If the discrepancy is over a threshold \(= K_{\text{factor}} \cdot S_{B_W}\) (e.g., \(K_{\text{factor}} = 4\)), declare cycle-slip. That is:

\[
\text{If } |B_W(s; k) - m_{B_W}(s; k - 1)| > K_{\text{factor}} S_{B_W}(s; k - 1),
\]

- Thence, cycle-slip.
- Update the mean and sigma values according to the equations:

\[
m_{B_W}(s; k) = \frac{k - 1}{k} m_{B_W}(s; k - 1) + \frac{1}{k} B_W(s; k)
\]
\[
S_{B_W}^2(s; k) = \frac{k - 1}{k} S_{B_W}^2(s; k - 1) + \frac{1}{k} (B_W(s; k) - m_{B_W}(s; k - 1))^2
\]

(4.24)

Note the \(S_{B_W}\) is initialised with an a priori \(S_0 = \lambda_w/2\).

\[B_W = L_W - P_N = \lambda_w N_W + \varepsilon\]
The Melbourne-Wübbena combination has a double benefit:

- The enlargement of the ambiguity spacing, thanks to the larger wavelength $\lambda_W = 80.4\text{cm}$.
- The noise is reduced by the narrow-lane combination of code measurement.

Nevertheless, in spite of these benefits, the performance is worse than in the previous carrier-phase-only based detector and it is used as a secondary test.
Exercises:

1) Show that $\Delta N_1 = 9$ and $\Delta N_2 = 7$
   produces jumps of few millimetres in the geometry-free combination.

2) Show that no jump happens in the geometry-free combination when
   $\Delta N_1 / \Delta N_2 = 77 / 60$. In particular when $\Delta N_1 = 77$ and $\Delta N_2 = 60$ the
   jump in the wide-lane combination is: $17 \lambda_w \square 15 m$

Hint: Consider the following relationships (from [RD-1]):

The effect of a jump in the integer ambiguities in terms of $\Delta N_1$, $\Delta N_2$
and $N_W$ is given next:

$$
\Delta \Phi_w, \Delta \Phi_f, \Delta \Phi_c \text{ variations}
$$

$$
\begin{align*}
\Delta \Phi_w &= \lambda_w \Delta N_w = \lambda_w \left( \Delta N_1 - \Delta N_2 \right) \\
\Delta \Phi_f &= \lambda_1 \Delta N_1 - \lambda_2 \Delta N_2 = (\lambda_2 - \lambda_1) \Delta N_1 + \lambda_2 \Delta N_w \\
\Delta \Phi_c &= \lambda_N \left( \frac{\Delta w}{\lambda_1} \Delta N_1 - \frac{\Delta w}{\lambda_2} \Delta N_2 \right) = \lambda_N \left( \Delta N_1 + \frac{\Delta w}{\lambda_2} \Delta N_W \right)
\end{align*}
$$

(4.20)
Example of Single frequency Cycle-slip detector

Input data: Code pseudorange ($P_i$) and carrier phase ($L_i$) measurements.

Output: [satellite (PRN), time, cycle-slip flag]

For each epoch ($k$)

For each tracked satellite ($s$):

- Declare cycle-slip when data hole greater than $tol_{\Delta t}$.
- If no data hole larger than $tol_{\Delta t}$, thence:
- Update an array with the last $N$ differences of

$$d(s; k) = L_1(s; k) - P_1(s; k)$$

That is: $[d(s; k - N), \ldots, d(s; k - 1)]$

- Compute the mean and sigma discrepancy over the previous $N$ samples $[k - N, \ldots, k - 1]$:

$$m_d(s; k - 1) = \frac{1}{N} \sum_{i=1}^{N} d(s; k - i)$$

$$m_d^2(s; k - 1) = \frac{1}{N} \sum_{i=1}^{N} d^2(s; k - i)$$

$$S_d(s; k - 1) = \sqrt{m_d^2(s; k - 1) - m_d^2(s; k - 1)}$$

(4.27)

- Compare the difference at the epoch $k$ with the mean value of differences computed over the previous $N$ samples window. If the value is over a threshold $n_T S_d$ (e.g., $n_T = 5$), declare cycle-slip
d

That is:

If $|d(s; k) - m_d(s; k - 1)| > n_T S_d(s; k - 1)$,

Thence, cycle-slip.

End

A cycle-slip is declared when a measurement differs from the mean bias value over a predefined threshold.

The detection is based on real-time computation of mean and sigma values of the differences ($d=L_1-P_1$) of the code pseudorange and carrier over a sliding window of $N$ samples (e.g. $N=100$).

Missed detection
This detector is affected by the code pseudorange noise and multipath as well as the divergence of the ionosphere. Higher sampling rate improves detection performance, but shortest jumps can still escape from this detector.

On the other hand, a minimum number of samples is needed for filter initialization in order to ensure a reliable value of sigma for the detection threshold.

More details, exercises and examples of software code implementation of these detectors can be found in [RD-1] and [RD-2].
Contents

1. Review of GNSS measurements.
2. Linear combinations of measurements.
3. Carrier cycle-slips detection.
4. Carrier smoothing of code pseudorange.
Carrier smoothing of code pseudorange

The noisy (but unambiguous) code pseudorange can be smoothed with the precise (but ambiguous) carrier. A simple algorithm is given next:

**Hatch filter:**

\[
\hat{P}(k) = \frac{1}{n} P(k) + \frac{n-1}{n} \left( \hat{P}(k-1) + L(k) - L(k-1) \right)
\]

where \( \hat{P}(1) = P(1) \) and

\[
\begin{align*}
&n = k; \quad k < N \\
&n = N; \quad k \geq N
\end{align*}
\]

This algorithm can be interpreted as a real-time alignment of the carrier phase with the code measurement:

\[
\hat{P}(k) = \frac{1}{n} P(k) + \frac{n-1}{n} \left( \hat{P}(k-1) + L(k) - L(k-1) \right) = L(k) + \left\{ P - L \right\}_{(k)}
\]
This algorithm can be interpreted as a real-time alignment of the carrier phase with the code measurement:

$$\hat{P}(k) = \frac{1}{n} P(k) + \frac{n-1}{n} \left( \hat{P}(k-1) + L(k) - L(k-1) \right) = L(k) + \langle P - L \rangle_{(k)}$$
Hatch filter: Carrier-smoothed code. N=100 epochs

\[
\hat{P}(k) = \frac{1}{n} P(k) + \frac{n-1}{n} \left( \hat{P}(k-1) + L(k) - L(k-1) \right) = L(k) + \langle P - L \rangle_k
\]
Hatch filter: Carrier-smoothed code. N=100 epochs

\[
\hat{P}(k) = \frac{1}{n} P(k) + \frac{n-1}{n} \left( \hat{P}(k-1) + L(k) - L(k-1) \right) = L(k) + \langle P - L \rangle_{(k)}
\]


**Code-carrier divergence: SF smoother**

Time varying ionosphere induces a bias in the single frequency (SF) smoothed code when it is averaged in the smoothing filter (Hatch filter).

Let:

\[
\begin{align*}
    P_1 &= \rho + I_1 + \varepsilon_1 \\
    L_1 &= \rho - I_1 + B_1 + \zeta_1
\end{align*}
\]

Where \( \rho \) includes all non dispersive terms (geometric range, clock offsets, troposphere) and \( I_1 \) represents the frequency dependent terms (ionosphere and DCBs). \( B_1 \) is the carrier ambiguity, which is constant along continuous carrier phase arcs and \( \varepsilon_1, \zeta_1 \) account for code and carrier multipath and thermal noise.

thence,

\[
P_1 - L_1 = 2I_1 - B + \varepsilon_1 \quad \Rightarrow 2I_1 : \text{Code-carrier divergence}
\]

Substituting \( P_1 - L_1 \) in Hatch filter equation

\[
\hat{P}(k) = L(k) + \langle P - L \rangle_{(k)} = \rho(k) - I_1(k) + B_1 + \langle 2I_1 - B_1 \rangle_{(k)} = \\
= \rho(k) + I_1(k) + 2\left( \langle I_1 \rangle_{(k)} - I_1(k) \right) = \hat{P}_1 = \rho + I_1 + \text{bias}_1 + \nu_1
\]

where, being the ambiguity term \( B_1 \) a constant bias, thence \( \langle B_1 \rangle_{(k)} \not\!\not\!\not\equal{} B_1 \), and cancels in the previous expression.  

where \( \nu_1 \) is the noise term after smoothing.
PRN03: C1 3600s smoothing and divergence of ionosphere

- C1 Raw
- C1 SF smoothed (3600)
- C1 DFree smoothed (3600)

N=3600 s

STEC PRN03 (shifted)

1.546*(L1-L2)
PRN03: C1 3600s smoothing and divergence of ionosphere

- C1 Raw
- C1 SF smoothed (3600)
- C1 DFree smoothed (3600)

meters

time (s)
Halloween storm

Data File: amc23030.03o_1Hz

\[ N=100 \text{ (i.e. filter smoothing time constant } \tau=100 \text{ sec).} \]
Carrier-smoothed pseudorange: DFree

Divergence-Free (Dfree) smoother:
With two frequency carrier measurements a combination of carriers with the same ionospheric delay (the same sign) as the code can be generated:

\[
L_{1,DF} = L_1 + 2\tilde{\alpha}_1 (L_1 - L_2) = \rho + I_1 + B_{1,DF} + \zeta_{1,DF}
\]

\[
\tilde{\alpha}_1 = \frac{f_2^2}{f_1^2 - f_2^2} = \frac{1}{\gamma - 1} = 1.545
\]

\[
\gamma = \left(\frac{77}{60}\right)^2
\]

With this new combination we have:

\[
P_1 = \rho + I_1 + \varepsilon_1
\]

\[
L_{1,DF} = \rho + I_1 + B_{1,DF} + \zeta_{1,DF}
\]

Thence,

\[
P_1 - L_{1,DF} = B_{1,DF} + \varepsilon_1
\]

\[\Rightarrow \text{No Code-carrier divergence!} \quad \Rightarrow \hat{P}_{1,DF} = \rho + I_1 + \nu_{12}\]

This smoothed code is immune to temporal gradients (unlike the SF smoother), being the same ionospheric delay as in the original raw code (i.e. \(I_1\)). Nevertheless, as it is still affected by the ionosphere, its spatial decorrelation must be taken into account in differential positioning.
PRN03: C1 3600s smoothing and divergence of ionosphere
Carrier-smoothed pseudorange: IFree

Ionosphere-Free (IFree) smoother:
Using both code and carrier dual-frequency measurements, it is possible to remove the frequency dependent effects using the ionosphere-free combination of code and carriers (PC and LC). Thence:

\[
P_{IFree} = P_C = \frac{\gamma P_1 - P_2}{\gamma - 1}; \quad L_{IFree} = L_C = \frac{\gamma L_1 - L_2}{\gamma - 1}
\]

\[
\gamma = \left(\frac{77}{60}\right)^2
\]

Thence,

\[
P_{IFree} = P_C = \rho + v_{IFree}
\]

This smoothed is based on the ionosphere-free combination of measurements, and therefore it is unaffected by either the spatial and temporal inospheric gradients, but has the disadvantage that the noise in amplified by a factor 3 (using the legacy GPS signals).
**Vertical range: [-5 : 5]**

PRN03: C1 3600s smoothing and divergence of ionosphere

- C1 Raw
- C1 SF smoothed (3600)
- C1 DFree smoothed (3600)

**Vertical range: [-15:15]**

Ionosphere-Free combination smoothing: 3600 seconds

- IFree raw
- IFree smth (3600s)
Exercise:

Justify that the ionosphere-free combination (PC) is (obviously) not affected by the code-carrier divergence, but it is 3 times noisier.
PRN03: C1 360s smoothing and divergence of ionosphere

N=360

PRN03: C1 100s smoothing and divergence of ionosphere

N=100

Ionosphere-Free combination smoothing: 360 seconds

N=360

Ionosphere-Free combination smoothing: 100 seconds

N=100
Halloween storm

Data File: amc23030.03o_1Hz

$N=100$ (i.e. filter smoothing time constant $\tau=100$ sec).
Contents

1. Review of GNSS measurements.
2. Linear combinations of measurements.
3. Carrier cycle-slips detection.
4. Carrier smoothing of code pseudorange.
Multipath

One or more reflected signals reach the antenna in addition to the direct signal. Reflective objects can be earth surface (ground and water), buildings, trees, hills, etc.

It affects both code and carrier phase measurements, and it is more important at low elevation angles.

**Code:** up to 1.5 chip-length \( \Rightarrow \) up to 450m for C1 [theoretically]
Typically: less than 2-3 m.

**Phase:** up to \( \lambda/4 \) \( \Rightarrow \) up to 5 cm for L1 and L2 [theoretically]
Typically: less than 1 cm
Exercise
Plot code and phase geometry-free combination for satellite PRN 15 of file 97jan09coco_r0.rnx and discuss the results.

Butterfly shape:
High multipath for low elevation rays (when satellite rises and sets)
\[ M_{PC} = PC - LC \]
\[ M_{Pc} = P_c - L_c \]
$M_{MW} = P_N - L_W$
\[ M_{MW} = P_N - L_W \]
After one year, the directions of the Sun and Aries coincide again, but the number of laps relative to the Sun (solar days) is one less than those relative to Aries (sidereal days).

\[
\frac{24h}{365.2422} \approx 3^m56^s
\]

Thus, a **sidereal day is shorter** than a solar day for about **3 m 56s**
Receiver and multipath noise

Barcelona, Spain: 2005-05-29
Receiver: NovAtel OEM3, Antenna: NovAtel 600 (Pinwheel)

GPS standalone (C1 code)

10,000 €
Receiver and multipath noise

GPS standalone (C1 code)

Same environment!

Barcelona, Spain: 2005 5 29  
Receiver: Trimble Lassen SK-II, Antenna: Compact Magnetic-Mount

East (meters)

North (meters)

100 €
References


Thank you!