Lecture 2

Satellite orbits and clocks computation and accuracy

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24 April 2014
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The GPS navigation message provides pseudo-Keplerian elements to compute satellite coordinates.
6 values are needed \((x, y, z, vx, vy, vz)\) to provide the position and velocity of a body. They can be mapped into the six Keplerian elements \((a, e, i, \Omega, \omega, V)\), which provides the “natural” representation of the orbit!
The orbit shape, orientation, and position in the orbit can be described using the following parameters:

- \((a, e, i, \Omega, \omega, \nu)\)
  - \(a\): semi-major axis
  - \(e\): eccentricity
  - \(i\): inclination
  - \(\Omega\): argument of perigee
  - \(\omega\): longitude of the ascending node
  - \(\nu\): true anomaly

- Perigee and focus
- Orbit shape
- Orbit orientation
- Position in the orbit

Additional parameters include:
- \(W\): angular momentum
- \(w\): mean anomaly
- \(V\): true anomaly
- \(\theta\): sidereal time
- \(\gamma\): vernal equinox
- \(G\): Greenwich meridian
Fictitious body moving at velocity \( n = 2\pi/P = \text{constant} \)

- **Mean anomaly** \( M(t) \)
- **Perigee**
- **True anomaly** \( V(t) \)

**Equations:**

\[
M(t) = n(t - T_0) ; \quad n = \frac{2\pi}{P} = \sqrt{\frac{\mu}{a^3}}
\]

\[
E(t) = M(t) + e \sin E(t)
\]

\[
V(t) = 2 \arctan \left[ \sqrt{\frac{1+e}{1-e}} \tan \frac{E(t)}{2} \right]
\]
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Due to the non-spherical nature of gravitational potential, the attraction of the Sun and Moon, the solar radiation pressure, etc., the true satellite path deviates from the elliptic orbit.

At any time an elliptical orbit tangent to the true path can be defined. This is the “osculating orbit”, whose Keplerian elements vary with time “t”:

\[ a(t), e(t), i(t), \Omega(t), \omega(t), V(t) \]
Different magnitudes of perturbation and their effects on GPS orbits

<table>
<thead>
<tr>
<th>Perturbation</th>
<th>Acceleration (m/s²)</th>
<th>Orbital effect in 3 hours</th>
<th>Orbital effect in 3 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central force (as a reference)</td>
<td>0.56</td>
<td>2 km</td>
<td>14 km</td>
</tr>
<tr>
<td>$J_2$</td>
<td>$5 \cdot 10^{-5}$</td>
<td>50–80 m</td>
<td>100–1500 m</td>
</tr>
<tr>
<td>Rest of the harmonics</td>
<td>$3 \cdot 10^{-7}$</td>
<td>5–150 m</td>
<td>1000–3000 m</td>
</tr>
<tr>
<td>Solar + Moon grav.</td>
<td>$5 \cdot 10^{-6}$</td>
<td>-</td>
<td>0.5–1.0 m</td>
</tr>
<tr>
<td>Tidal effects</td>
<td>$1 \cdot 10^{-9}$</td>
<td>5–10 m</td>
<td>100–800 m</td>
</tr>
<tr>
<td>Solar rad. pressure</td>
<td>$1 \cdot 10^{-7}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Session 3.1, Ex8c: Arg. of ascending node

- Central body
- Central body + J2
- Central body + J2 + SM

Degrees

Time (s)

0 10000 20000 30000 40000 50000 60000 70000 80000 90000
Session 3.1, Ex8c: Inclination

- Central body
- Central body + J2
- Central body + J2 + SM
Session 3.1, Ex8c: Argument of Perigee

Central body

Central body + J2

Central body + J2 + SM
Calculation of osculating orbital elements from position and velocity (rv2osc.f)

\[(x, y, z, v_x, v_y, v_z) \Rightarrow (a, e, i, \Omega, \omega, M)\]

\[\vec{c} = \vec{r} \times \vec{v} \implies p = \frac{c^2}{\mu} \implies p\]
\[v^2 = \mu(2/r - 1/a) \implies a\]
\[p = a \ (1 - e^2) \implies e\]

\[\vec{c} = c\vec{S} \implies \Omega = \arctan(-c_x/c_y); \quad i = \arccos(c_z/c) \implies \Omega, i\]

\[
\begin{pmatrix}
 x \\
 y \\
 z
\end{pmatrix} = R
\begin{pmatrix}
 r \cos(V) \\
 r \sin(V) \\
 0
\end{pmatrix}
= r
\begin{pmatrix}
 \cos \Omega \cos(\omega + V) - \sin \Omega \sin(\omega + V) \cos i \\
 \sin \Omega \cos(\omega + V) + \cos \Omega \sin(\omega + V) \cos i \\
 \sin(\omega + V) \sin i
\end{pmatrix} \implies \omega + V
\]

\[r = \frac{p}{1 + e \cos(V)} \implies \omega, V\]

\[\tan(E/2) = \left(\frac{1 - e}{1 + e}\right)^{1/2} \tan(V/2) \quad ; \quad M = E - e \sin E \implies M\]
Calculation of position and velocity from osculating orbital elements (osc2rv.f)

\[(a, e, i, \Omega, \omega, T, t) \Rightarrow (x, y, z, v_x, v_y, v_z)\]

\[
t \quad \Rightarrow \quad M \quad \Rightarrow \quad E \quad \Rightarrow \quad (r, V)
\]

\[M = n(t - T) \quad \quad M = E - e \sin E \quad \quad r = a(1 - e \cos E)
\]
\[
\tan(V/2) = \left(\frac{1 + e}{1 - e}\right)^{1/2} \tan(E/2)
\]

\[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
= R
\begin{pmatrix}
r \cos(V) \\
r \sin(V) \\
0
\end{pmatrix};
\begin{pmatrix}
v_x \\
v_y \\
v_z
\end{pmatrix}
= \frac{na^2}{r} \left\{ \vec{Q}(1 - e^2)^{1/2} \cos E - \vec{P} \sin E \right\}
\]

Where:

\[
R = R_3(-\Omega)R_1(-i)R_3(-\omega) =
\]
\[
= \begin{pmatrix}
\cos \Omega & -\sin \Omega & 0 \\
\sin \Omega & \cos \Omega & 0 \\
0 & 0 & 1
\end{pmatrix}
\cdot
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos i & -\sin i \\
0 & \sin i & \cos i
\end{pmatrix}
\cdot
\begin{pmatrix}
\cos \omega & -\sin \omega & 0 \\
\sin \omega & \cos \omega & 0 \\
0 & 0 & 1
\end{pmatrix}
\]
\[
= \begin{pmatrix}
P_x & Q_x & S_x \\
P_y & Q_y & S_y \\
P_z & Q_z & S_z
\end{pmatrix}
= [\vec{P} \quad \vec{Q} \quad \vec{S}]
\]
Exercise: Orbital elements variation:


a) Use program "rv2osc" to compute the instantaneous orbital elements (X, Y, Z, Vx,Vy,Vz) \(\rightarrow\) (a, e, i, \(\Omega\), \(\omega\), V)

b) Plot the orbital elements in function of time to show their variation: \(a(t), e(t), i(t), \Omega(t), \omega(t), V(t)\)

c) Compare with the broadcast orbital elements

Solution:

a) cat 1995-10-18.eci|rv2osc> orb.dat
b) See the following plots
Session 3.1, Ex6a: Argument of Perigee

Session 3.1, Ex6a: Mean Anomaly
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GPS navigation message

One Master Frame includes All 25 pages of Subframes #4 and #5 \( \Rightarrow 25 \times 30s = 12.5 \text{ min} \)
Subframe 1 contains information about the parameters to be applied to satellite clock status for its correction. These values are polynomial coefficients that allow time onboard to be converted to GPS time. The subframe also contains information on satellite health condition.

Subframes 2 and 3 contain satellite ephemerides.

Subframe 4 provides ionospheric model parameters (in order to adjust for ionospheric refraction), UTC information, part of the almanac, and indications whether the A/S is activated or not (which transforms the P code into encrypted Y code).

Subframe 5 contains data from the almanac and on constellation status. It allows rapid identification of the satellite from which the signal comes. A total of 25 frames are needed to complete the almanac.
### Ephemeris in navigation message

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( IODE )</td>
<td>Series number of ephemerides data</td>
</tr>
<tr>
<td>( t_{oe} )</td>
<td>Ephemerides reference epoch</td>
</tr>
<tr>
<td>( \sqrt{a} )</td>
<td>Square root of semi-major axis</td>
</tr>
<tr>
<td>( e )</td>
<td>Eccentricity</td>
</tr>
<tr>
<td>( M_o )</td>
<td>Mean anomaly at reference epoch</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Argument of perigee</td>
</tr>
<tr>
<td>( i_o )</td>
<td>Inclination at reference epoch</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>Ascending node’s right ascension</td>
</tr>
<tr>
<td>( \Delta n )</td>
<td>Mean motion difference</td>
</tr>
<tr>
<td>( \dot{i} )</td>
<td>Rate of inclination angle</td>
</tr>
<tr>
<td>( i )</td>
<td>Rate of node’s right ascension</td>
</tr>
<tr>
<td>( c_{uc}, c_{us} )</td>
<td>Latitude argument correction</td>
</tr>
<tr>
<td>( c_{rc}, c_{rs} )</td>
<td>Orbital radius correction</td>
</tr>
<tr>
<td>( c_{ic}, c_{is} )</td>
<td>Inclination correction</td>
</tr>
</tbody>
</table>

In order to calculate WGS84 satellite coordinates, you should apply the following algorithm [GPS/SPS-SS, table 2-15] (see in the book FORTRAN subroutine orbit.f)
### RINEX ephemeris file

<table>
<thead>
<tr>
<th>G1</th>
<th>GPS</th>
<th>RINEX VERSION/TYPE</th>
<th>NAVIGATION DATA</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### GAGE BROADCAST EPHEMERIS FILE

<table>
<thead>
<tr>
<th>ION ALPHA</th>
<th>ION BETA</th>
<th>DELTA.UTC: A0,A1,T,W</th>
<th>LEAP SECONDS</th>
<th>END OF HEADER</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1.7695E-08 +2.2352E-08 -1.1921E-07 -1.1921E-07</td>
<td>+1.1878E+05 +1.4746E+05 -1.3107E+05 -3.2768E+05</td>
<td>1064</td>
<td>405504</td>
<td></td>
</tr>
</tbody>
</table>

| 03 | 00 | 5 30 10 0 | 40.0+7.855705916882E-06+3.524291969370E-12+0.000000000000E+00 | Mo |

| +1.010000000000E+02+6.500000000000E+01+5.456298524109E-09+5.53028558107E-01 | e, √a |

| +3.475695848465E-06+1.308503560722E-03+2.641230821609E-06+5.153678266525E+03 | TOE, Ω |

| +2.083000000000E+05+1.117587089539E-08+7.472176136643E-01-1.862645149231E-09 | i0, ω |

| +9.412719852649E-01+3.163750000000E+02+1.125448382894E+00-8.826796182859E-09 | TGD |

| +1.239337382719E-10+1.000000000000E+00+1.064000000000E+03+0.000000000000E+00 |  |

| +4.000000000000E+00+0.000000000000E+00-4.190951585770E-09+6.130000000000E+02 |  |

| +2.044980000000E+05+0.000000000000E+00+0.000000000000E+00+0.000000000000E+00 |  |

| 06 | 00 | 5 30 10 0 | 0.0+1.636799424887E-06+0.000000000000E+00+0.000000000000E+00 |  |

| +6.000000000000E+01+5.100000000000E+01+5.198073527168E-09-5.601816471398E-01 |  |

| +2.635642886162E-06+6.763593177311E-03+2.468004822731E-06+5.153726325989E+03 |  |

| +2.068000000000E+05+1.862645149231E-08+7.894129138508E-01+8.19563656616E-08 |  |

| +9.487675576456E-01+3.229687500000E+02-2.409256713064E+00-8.734292400447E-09 |  |

| +4.714481929846E-11+1.000000000000E+00+1.064000000000E+03+0.000000000000E+00 |  |
3.1. Computation of satellite coordinates from navigation message (orbit.f)

- Computation of $t_k$ time since ephemerids reference epoch $t_{oe}$ ($t$ and $t_{oe}$ are given in GPS seconds of week):

$$t_k = t - t_{oe}$$

- Computation of mean anomaly $M_k$ for $t_k$:

$$M_k = M_0 + \left( \frac{\sqrt{\mu}}{\sqrt{a^3}} + \Delta n \right) t_k$$

- Iterative resolution of Kepler’s equation in order to compute eccentric anomaly $E_k$:

$$M_k = E_k - e \sin E_k$$

- Calculation of true anomaly $v_k$:

$$v_k = \arctan\left( \frac{\sqrt{1 - e^2} \sin E_k}{\cos E_k - e} \right)$$

- Computation of latitude argument $u_k$ from perigee argument $W$, true anomaly $v_k$ and corrections $c_{uc}$ and $c_{us}$:

$$u_k = \omega + v_k + c_{uc} \cos 2(\omega + v_k) + c_{us} \sin 2(\omega + v_k)$$
• Computation of radial distance $r_k$ taking into consideration corrections $c_{rc}$ and $c_{rs}$:

$$r_k = a \left(1 - 2 \cos E_k\right) + c_{rc} \cos 2\left(\omega + v_k\right) + c_{rs} \sin 2\left(\omega + v_k\right)$$

• Calculation of orbital plane inclination $i_k$ from inclination $i_o$ at reference epoch $t_{oe}$ and corrections $c_{ic}$ and $c_{is}$:

$$i_k = i_o + it_k + c_{ic} \cos 2\left(\omega + v_k\right) + c_{is} \sin 2\left(\omega + v_k\right)$$

• Computation of ascending node longitude $\Omega_k$ (Greenwich), from longitude $\Omega_0$ at start of GPS week, corrected from apparent variation of sidereal time at Greenwich between start of week and and reference time $t_k = t - t_{oe}$ and also corrected from change of ascending node longitude since reference epoch $t_{oe}$:

$$\Omega_k = \Omega_0 + \left(\Omega - \omega_E\right)t_k - \omega_E t_{oe}$$

• Calculation of coordinates in CTS system, applying three rotations (around $u_k$, $i_k$, $\Omega_k$):

$$\begin{bmatrix}
X_k \\
Y_k \\
Z_k
\end{bmatrix} = R_3(-\Omega_k)R_1(-i_k)R_3(-u_k)
\begin{bmatrix}
r_k \\
0 \\
0
\end{bmatrix}$$
Conventional Terrestrial System (CTS):

Earth Centred, Earth-Fixed (ECEF) System

the reference system rotates with Earth.
Selective Availability (S/A): Intentional degradation of satellite clocks and broadcast ephemeris. (from 25 March, 1990)

GPS Before and After S/A was switched off

ANALYSIS NOTES
- Data taken from Overlook PAN Monitor Station, equipped with Trimble SVeeSix Receiver
- Single Frequency Civil Receiver
- Four Satellite Position Solution at Surveyed Benchmark
- Data presented is raw, no smoothing or editing
Session 3.2, Ex1a: GPS Broadcast - Precise: Along Track error

Session 3.2, Ex1a: GPS Broadcast - Precise: Cross Track error

Session 3.2, Ex1a: GPS Broadcast - Precise: Radial error

Session 3.2, Ex1a: GPS Broadcast - Precise: Clock error

SA=off

Master of Science in GNSS

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3.2 Computation of satellite coordinates from precise products.

Precise orbits for GPS satellites can be found on the International GNSS Service (IGS) server http://igscb.jpl.nasa.gov

Orbits are given by \((x, y, z)\) coordinates with a sampling rate of 15 minutes. The satellite coordinates between epochs can be computed by polynomial interpolation. A 10th-order polynomial is enough for a centimetre level of accuracy with 15 min data.

\[
P_n(x) = \sum_{i=1}^{n} y_i \frac{\prod_{j \neq i} (x - x_j)}{\prod_{j \neq i} (x_i - x_j)}
\]

\[
= y_1 \frac{x - x_2}{x_1 - x_2} \cdots \frac{x - x_n}{x_1 - x_n} + \cdots
\]

\[
+ y_i \frac{x - x_1}{x_i - x_1} \cdots \frac{x - x_i - 1}{x_i - x_i - 1} \frac{x - x_i + 1}{x_i - x_i + 1} \cdots \frac{x - x_n}{x_i - x_n} + \cdots
\]

\[
+ y_n \frac{x - x_1}{x_n - x_1} \cdots \frac{x - x_n}{x_n - x_n - 1}
\]
IGS orbit and clock products (for PPP):
Discrepancy between the different centres
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GPS Satellite Clock computation: Broadcast message

\[ d^{sat} = a_0 + a_1(t-t_0) + a_2(t-t_0)^2 \]

### 2. Navigation Data

- **PRN**: 14
- **YY MM DD H M S**: 95 10 18 00 51 44.0
- **NAVIGATION DATA**: 1.129414886236D-05
- **a0**: 1.136868377216D-13
- **a1**: 0.000000000000D+00
- **a2**: 0.000000000000D+00

### RINEX Version / Type

- **srx/v1.8.1.4**: BAI
- **PGM / RUN BY / DATE**: 95/10/19 03:18:35
- **COMMENT**:

```plaintext
2444431.2031 -4428688.6270 3875750.1442
```

### GPS Satellite Clock computation: Broadcast message

\[ t_0 = \frac{a_0 + a_1(t-t_0) + a_2(t-t_0)^2}{a_2} \]
Computation of satellite clocks from precise products

Precise clocks for GPS satellites can be found on the International GNSS Service (IGS) server [http://igscb.jpl.nasa.gov](http://igscb.jpl.nasa.gov)

They are providing precise orbits and clock files with a sampling rate of 15 min, as well as precise clock files with a sample rate of 5 min and 30 s in SP3 format.

Some centres also provide GPS satellite clocks with a 5 s sampling rate, like the les obtained from the Crustal Dynamics Data Information System (CDDIS) site.

Stable clocks with a sampling rate of 30 s or higher can be interpolated with a first-order polynomial to a few centimetres of accuracy. Clocks with a lower sampling rate should not be interpolated, because clocks evolve as random walk processes.
IGS orbit and clock products (for PPP):
Discrepancy between the different centres

Exercise:
Show that a common error on all satellites does not affect user positioning.
Session 3.2, Ex7a: Precise 300s clock interpolation error

metres

0 10000 20000 30000 40000 50000 60000 70000 80000 90000
time (s)
Session 3.2, Ex7b: Precise 30s clock interpolation error
Session 3.2, Ex8: Precise 300s clock interpolation error (SA=on/off)

- Red dots: All sat
- Blue line: PRN01

Vertical axis: metres
Horizontal axis: time (s)
### IGS Precise orbit and clock products: RMS accuracy, latency and sampling

<table>
<thead>
<tr>
<th>Products (delay)</th>
<th>Broadcast (real time)</th>
<th>Ultra-rapid Predicted (real time)</th>
<th>Observed (3–9 h)</th>
<th>Rapid (17–41 h)</th>
<th>Final (12–18 d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbit GPS (sampling)</td>
<td>~100 cm ( ~2 h)</td>
<td>~5 cm (15 min)</td>
<td>~3 cm (15 min)</td>
<td>~2.5 cm (15 min)</td>
<td>~ 2.5 cm (15 min)</td>
</tr>
<tr>
<td>Glonass (sampling)</td>
<td>~5 ns (daily)</td>
<td>~3 ns (15 min)</td>
<td>~150 ps (15 min)</td>
<td>~75 ps (5 min)</td>
<td>~75 ps (30 s)</td>
</tr>
<tr>
<td>Clock GPS (sampling)</td>
<td>Glonass (sampling)</td>
<td>~5 ns (daily)</td>
<td>~3 ns (15 min)</td>
<td>~150 ps (15 min)</td>
<td>~75 ps (30 s)</td>
</tr>
</tbody>
</table>

http://igscb.jpl.nasa.gov/components/prods.html
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Ephemeris Errors and Geographic decorrelation

True position

Satellite location error

Position from broadcast ephemeris

Differential range error due to satellite orbit error

\[ \delta \rho = \vec{\varepsilon} \cdot \frac{\vec{\rho}_{\text{user}}}{\rho_{\text{user}}} - \vec{\varepsilon} \cdot \frac{\vec{\rho}_{\text{ref}}}{\rho_{\text{ref}}} \]

A conservative bound:

\[ \delta \rho < \frac{b}{\rho} \]

with a baseline \( b = 20 \text{km} \)

\[ \delta \rho < \frac{20}{20000} \varepsilon = \frac{1}{1000} \varepsilon \]
Satellite location error $\varepsilon$

True position

Position from broadcast ephemeris

Reference Station

User

Baseline: $b$

Range error from CREU and EBRE

Differential range error from between CREU and EBRE

$\delta \rho = \varepsilon \cdot \frac{\hat{\rho}_{\text{user}}}{\rho_{\text{user}}} - \varepsilon \cdot \frac{\hat{\rho}_{\text{ref}}}{\rho_{\text{ref}}}$

288 km of baseline

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**Range error from CREU and EBRE**

\[ \delta \rho = \mathbf{\varepsilon} \cdot \rho_{user} - \mathbf{\varepsilon} \cdot \rho_{ref} \]

**288 km of baseline**
Errors over the hyperboloid (i.e. $\rho_B - \rho_A = ctt$) will not produce differential range errors.

The highest error is given by the vector $\hat{u}$, orthogonal to the hyperboloid and over the plain containing the baseline vector $\hat{b}$ and the LoS vector $\hat{\rho}$.

Note: Being the baseline $b$ much smaller than the distance to the satellite, we can assume that the LoS vectors from A and B receives are essentially identical to $r$.

Let $\delta \rho = \delta (\rho_B - \rho_A) = 2 \delta a = 2 \frac{\partial a}{\partial \xi} \delta \xi = 2 \frac{\partial a}{\partial \phi} \frac{\partial \phi}{\partial \xi} \delta \phi = -b \sin \phi \frac{\partial \phi}{\partial \xi} \delta \phi \\
\approx -b \sin \phi \frac{1}{\rho} \delta \phi 

Note: $\xi \perp \hat{\rho} \Rightarrow \xi = \rho \delta \phi$

Let $\xi \parallel \hat{u}$ parallel to vector $\hat{u}$

Let $\epsilon \equiv \xi \parallel$ 

Thence:

$$\delta \phi = - \frac{b \sin \phi}{\rho} \xi \cdot \hat{u} = - \xi \cdot \left( \sin \phi \hat{u} \right) \frac{b}{\rho}$$

$$= - \xi^T \left( I - \hat{\rho} \cdot \hat{\rho}^T \right) \frac{b}{\rho}$$

Where: $b = b \hat{b}$ is the baseline vector
**ORBIT TEST:**
Broadcast orbits
Along-track Error (PRN17)

**PRN17:**
Doy=077, Transm. time: 64818 sec
**Objectives**

- **Orbit error** $\mathbf{\varepsilon}$
- **Range error**
- **Differential range error**

**Baseline:** $b=31.3 \text{ km}$

**Equation:**

$$\delta \rho = -\mathbf{\varepsilon}^T (\mathbf{I} - \mathbf{\rho} \cdot \mathbf{\rho}^T) \frac{\mathbf{b}}{\rho}$$

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**Legend:**

- **Rad**
- **Alon**
- **Cross**
Exercise:

Justify that clock errors completely cancel in differential positioning.
ERRORS on the Signal

- **Space Segment Errors:**
  - Clock errors
  - Ephemeris errors

- **Propagation Errors**
  - Ionospheric delay
  - Tropospheric delay

- **Local Errors**
  - Multipath
  - Receiver noise

- Strong spatial correlation
- Weak spatial correlation
- No spatial correlation
References


Thank you